Robust bipartite output regulation of linear uncertain multi-agent systems under observer-based protocols

Jiashuo Liu[1], Cui-Qin Ma[1], Yun-Bo Zhao[2], and Yu Kang[2]

***Abstract*—Robust bipartite output regulation of linear uncertain multi-agent systems is studied over a signed digraph. Since only parts of agents have access to the information of the exosystem, a distributed observer is introduced to estimate the exosystem state for each agent. Then, a distributed control protocol is proposed based on the internal model method and observer for the exosystem. By exploiting matrix analysis and algebraic graph theory, sufficient conditions for achieving robust bipartite output regulation are given. It is shown that the multi-agent system being structurally balanced and the augmented multi-agent system having a spanning tree with the exosystem being the root are the communication topology conditions for ensuring robust bipartite output regulation. Finally, the correctness of the results is validated by an example.**

***Index Terms*—Robust bipartite output regulation, linear multi-agent systems, signed digraph, structurally balanced.**

I. Introduction

OUTPUT regulation (OR), one of the central issues in coordination control for multi-agent systems (MASs), is to design a controller for each agent using interactions among agents such that the agent output can track the reference signal generated by the exosystem while preserving the closed-loop system stability. Generally, the exosystem can be regarded as a virtual leader of the MAS, and OR in this sense has also been seen as an extension of the leader-following consensus problems ([1], [2]). OR has appealed to extensive attention from the research community in recent years ([3]–[11]). In [3], for linear MASs with each agent having a distributed observer for the exosystem, a distributed state feedback controller with observers was proposed and sufficient conditions for ensuring OR were established. Then, the distributed controller with global information of communication topology in [3] was improved in [4] to a fully distributed adaptive controller over digraphs, and OR with asymptotical convergence in [3] was extended in [5] to the finite-time stable manner. Moreover, OR in the presence of actuator faults and DoS attacks was studied in [6]. Note that the agent dynamic parameters in [3]–[6] are all assumed to be completely known. However, in reality, agents are easily subject to external disturbances and their parameters are commonly uncertain. For uncertain MASs, OR has been thoroughly studied ([7]–[9]). To name a few, in [7], for linear uncertain MASs, depending on an internal model approach, distributed state feedback and output feedback protocols were given for agents, respectively. For guaranteeing robust OR, necessary and sufficient conditions regarding the communication graph of the MAS were proposed. Further, the robust OR problem in [7] was generalized in [8] to the scenarios with communication and input time-delays.

The aforementioned works focus only on cooperative interactions among agents, but not antagonism that is indispensable in the real world. By modeling the interactions among agents as a signed graph with positive and negative edges for cooperation and antagonism, respectively ([12]–[14]), bipartite OR problem can be properly modeled and has attracted much attention ([15]–[20]). For example, in a two-party scenario, one supports only its own party. Hence, party members are cooperative in their own party but are antagonistic in between. This leads to bipartite OR naturally, characterized by exactly opposite objectives of agents. The bipartite OR problem exists for both certain ([15]–[19]) and uncertain systems ([20]). For the former, in [15], the relation between OR and bipartite OR for linear MASs was investigated, showing their equivalence in some sense. Subsequently, in [16], under the assumption that only parts of agents can get information of the exosystem, distributed observers were designed, based on which distributed output feedback protocols were constructed for linear MAS via an internal model principle to reach bipartite OR. Besides, bipartite OR problem for heterogeneous MASs was considered in [17], [18]. However, robust bipartite OR for uncertain systems has not been well studied to date.

In this paper, robust bipartite OR of linear uncertain MASs is studied under a signed digraph. Each agent is described by a linear system with uncertain parameters, subject to different influence by the exosystem. Unlike the results in [20], here each agent cannot utilize the output of the exosystem for its controller design, even though it has communication link with the exosystem. Only parts of agents can synthesize controllers using the state of the exosystem. So, to estimate the exosystem state, a distributed observer is introduced, and it is proved that the state estimation error of each observer converges to zero asymptotically. In contrast with existing results on cooperative OR problem, there exist both cooperation and antagonism for robust bipartite OR problem under signed digraphs. Thus, the output error between agents is not merely the difference of the agent outputs and might be the sum. This brings difficulties in protocol design and closed-loop system analysis.

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To overcome the difficulties, an observer-based distributed protocol is proposed with the help of an internal model principle. By exploiting theories of matrix analysis and related lemmas, properties regarding the convergence of the closed-loop system are fully characterized. Sufficient conditions for solving robust bipartite OR problem are obtained for linear uncertain MASs under signed digraphs.

The contributions of the present work are threefold. First, the problem setting considers the joint impact of system uncertainty and unknown exosystem states on bipartite OR. Second, a distributed control protocol on the basis of observer and the internal model principle is proposed for each agent. Finally, sufficient conditions to achieve robust bipartite OR are given for linear uncertain MASs.

The remainder is organized as follows. Relevant results concerning graph theory and problems of robust bipartite OR are discussed in Section II. The main results for robust bipartite OR problem are presented in Section III. We give an example to substantiate the analysis in Section IV and conclude the paper in Section V.

Notations: I_n is a n-dimensional identity matrix. $\mathbf{1}_N$ denotes a column vector with all elements being 1. $\lambda_i(A)$ represents the ith eigenvalue of matrix A. diag$(\cdot)$ means a diagonal matrix. Re$(\cdot)$ represents the real part. vec$(B) : \mathbb{R}^{n\times m} \to \mathbb{R}^{nm}$ represents vector function of matrix $B \in \mathbb{R}^{n\times m}$. sgn$(\cdot)$ denotes a sign function.

II. Preliminaries

A. Graph Theory

For a signed digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with N agents, $\mathcal{V} = \{1, \cdots, N\}$, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$. If there exists an edge from k to j, i.e., $(k, j) \in \mathcal{E}$, then $a_{jk} \neq 0$. In particular, $a_{jk} > 0$ means cooperation between j and k while $a_{jk} < 0$ means antagonism. In this paper, we always assume that $a_{kk} = 0$ and $a_{kj}a_{jk} \geq 0$. Let $\mathcal{N}_i = \{k|(k, i) \in \mathcal{E}\}$ be agent i's neighbour set. Laplacian matrix of $\mathcal{G}$ is given by $\mathcal{L} = (l_{ij})$, in which $l_{ii} = \sum_{k=1}^{N} |a_{ik}|$, and $l_{ij} = -a_{ij}$, $i \neq j$. If there is a directed path from one node to all the other nodes in a digraph, then the digraph contains a spanning tree with this node as its root.

$\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is structurally balanced if $\mathcal{V}$ can be divided into two disjoint subsets $\mathcal{V}_1, \mathcal{V}_2$ such that $a_{kl} \geq 0, \forall k, l \in \mathcal{V}_t\,(t \in \{1, 2\})\,; a_{kl} \leq 0, \forall k \in \mathcal{V}_t, \forall l \in \mathcal{V}_s, (t \neq s, t, s \in \{1, 2\})$ and structurally unbalanced, otherwise.

B. System Models

The considered MAS contains N agents, with agent i being modeled as follows,

$$\begin{aligned} \dot{x}_i &= \bar{A}x_i + \bar{B}u_i + \bar{E}_i v \\ y_i &= \bar{C}x_i, \quad i = 1, \cdots, N, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, $y_i \in \mathbb{R}^p$ are the state, input and output of agent i, respectively. $v \in \mathbb{R}^q$, the state of the exosystem, represents a reference signal and disturbance, which satisfies

$$\begin{aligned} \dot{v} &= Sv \\ y_0 &= Fv, \end{aligned} \tag{2}$$

where $y_0 \in \mathbb{R}^p$ is the reference output, S and F are known constant matrices. In addition, $\bar{A}, \bar{B}, \bar{C}, \bar{E}_i$ are uncertain matrices with compatible dimensions, and

$$\begin{aligned} &\bar{A} = A + \Delta A, \bar{B} = B + \Delta B, \bar{C} = C + \Delta C, \\ &\bar{E}_i = E_i + \Delta E_i (i = 1, \cdots, N), \end{aligned}$$

where A, B, C, E_i are known matrices and ΔA, ΔB, ΔC, ΔE_i represent unknown uncertain matrices.

For convenience, we define uncertain vector

$$\begin{aligned} \Delta =& \big((\mathrm{vec}\,(\Delta A))^T, (\mathrm{vec}\,(\Delta B))^T, (\mathrm{vec}\,(\Delta C))^T, \\ & (\mathrm{vec}\,(\Delta E_1))^T, \cdots, (\mathrm{vec}\,(\Delta E_N))^T\big)^T \in \mathbb{R}^{n(n+m+p+Nq)}. \end{aligned}$$

When $\Delta = 0$, system (1) is a nominal system.

We regard the exosystem as a virtual leader, labelled 0. Let $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ be an augmented signed digraph, where $\bar{\mathcal{V}} = \mathcal{V} \cup \{0\}$, $\bar{\mathcal{A}} = [a_{ij}]$, $i, j \in \bar{\mathcal{V}}$. Furthermore, $a_{i0} \neq 0$ if agent i can obtain information from leader 0 and $a_{i0} = 0$, otherwise. Let $T = \mathrm{diag}(a_{10}, a_{20}, \cdots, a_{N0}) \in \mathbb{R}^{N\times N}$ be the pinning matrix between leader and agents. It is worth noting that in this paper the pinning weight a_{i0} can be negative, without having to be all nonnegative. $\bar{\mathcal{L}}$ denotes Laplacian matrix of $\bar{\mathcal{G}}$. Then,

$$\overline{\mathcal{L}} = \begin{pmatrix} 0 & 0 \\ -T\mathbf{1}_N & H \end{pmatrix},$$

where $H = \mathcal{L} + \bar{T}$, $\bar{T} = \mathrm{diag}(|a_{10}|, |a_{20}|, \cdots, |a_{N0}|)$.

For further analysis, we introduce the following assumptions.

(A1) $\mathrm{Re}(\lambda_i(S)) \geq 0, i = 1, \cdots, q$.
(A2) (A, B) is stabilizable.
(A3) $\forall \lambda \in \varrho(S)$, $\mathrm{rank}\begin{pmatrix} A - \lambda I & B \\ C & 0 \end{pmatrix} = n+p$, where $\varrho(S)$ is the spectrum of S.
(A4) $\mathcal{G}$ is structurally balanced and $\bar{\mathcal{G}}$ has a spanning tree with virtual leader 0 as its root.

Remark 1: (A1)-(A3) are common assumptions in OR problems ([21]). They are employed to guarantee the solvability of robust bipartite OR. Assumption (A4) is the requirement on communication topology for achieving OR. Unlike most existing results, where $\bar{\mathcal{G}}$ is supposed to be structurally balanced, here only $\mathcal{G}$ is required to be structurally balanced.

If Assumption (A4) holds, $\mathcal{G}$ is structurally balanced. Then, define output error of agent i as

$$e_i(t) = y_i(t) - d_i y_0(t), \tag{3}$$

where $d_i = 1, i \in \mathcal{V}_1$; $d_i = -1, i \in \mathcal{V}_2$.

Definition 1 ([22]): For linear uncertain MAS (1) and exosystem (2), if there exists a control protocol for each agent, such that

(i) the system matrix of the nominal closed-loop system is Hurwitz;
(ii) there is an open neighborhood $\mathcal{U}$ of $\Delta = 0$, such that $\forall \Delta \in \mathcal{U}$ and for any initial condition, $\lim_{t\to\infty} e_i(t) = 0, i = 1, \cdots, N$,

then linear uncertain MAS (1) is said to achieve robust bipartite OR.

The main aim of this paper is to develop a protocol for each agent in MAS (1) such that the agent output can track the exosystem output or its opposite one, i.e., robust bipartite OR. To this end, we present the following assumption and lemmas.

Definition 2 ([22]): (Π_1, Π_2) incorporates the minimum p-copy internal model of matrix S, if $\Pi_1 = \text{diag}\underbrace{(\Phi, \ldots, \Phi)}_{p\text{-tuple}}$, $\Pi_2 = \text{diag}\underbrace{(\chi, \ldots, \chi)}_{p\text{-tuple}}$, where Φ is a constant square matrix whose characteristic polynomial equals the minimal polynomial of S, and χ is a constant column vector such that (Φ, χ) is controllable.

Before listing the lemmas, another assumption is needed.

Assumption (A5): (Π_1, Π_2) incorporates the minimum p-copy internal model of S.

Lemma 1 ([22]): If Assumptions (A1)-(A3) and (A5) hold, then (M, N) is stabilizable, where

$$M = \begin{pmatrix} A & 0 \\ \Pi_2 C & \Pi_1 \end{pmatrix}, \quad N = \begin{pmatrix} B \\ 0 \end{pmatrix}.$$

Lemma 2 ([22]): Suppose Assumptions (A1) and (A5) hold. If $A_c = \begin{pmatrix} \hat{A} & \hat{B} \\ \Pi_2 \hat{C} & \Pi_1 \end{pmatrix}$ is Hurwitz, then $\forall\, \hat{E}, \hat{F}$,

$$\begin{cases} QS = \hat{A}Q + \hat{B}R + \hat{E} \\ RS = \Pi_1 R + \Pi_2(\hat{C}Q - \hat{F}) \end{cases}$$

has a unique solution Q and R. Furthermore, Q and R satisfy $0 = \hat{C}Q - \hat{F}$.

III. Main Results

Since only parts of agents have access to information of the virtual leader, a distributed control protocol based on observer is proposed for the i-th agent:

$$u_i = K_1\big(\sum_{j\in\mathcal{N}_i} a_{ij}(\text{sgn}(a_{ij})x_i - x_j) + |a_{i0}|x_i\big) + K_2 z_i, \tag{4}$$

$$\begin{aligned} \dot{z}_i = \;& \Pi_1 z_i + \Pi_2\big(\textstyle\sum_{j\in\mathcal{N}_i} a_{ij}(\text{sgn}(a_{ij})y_i - y_j) + |a_{i0}|y_i\big) \\ & -\Pi_2\big(\textstyle\sum_{j\in\mathcal{N}_i} a_{ij}(\text{sgn}(a_{ij})F\xi_i - F\xi_j) + |a_{i0}|F\xi_i\big), \end{aligned} \tag{5}$$

$$\dot{\xi}_i = S\xi_i - \gamma\big(\sum_{j\in\mathcal{N}_i} a_{ij}(\text{sgn}(a_{ij})\xi_i - \xi_j) + |a_{i0}|\,(\xi_i - d_i v)\big), \tag{6}$$

where $z_i \in \mathbb{R}^{n_z}$ is the state of the internal model constructed for agent i, $\xi_i \in \mathbb{R}^q$ is the state estimation of the exosystem, F is introduced in (2). $\gamma > 0$ is the gain coefficient, and Π_1, Π_2, K_1, K_2 are gain matrices.

Remark 2: Protocol (4) is designed based on the internal model principle, which can deal with parameter uncertainty in system (1) effectively with more robustness. Moreover, protocol (4) only uses information of agent i and its neighbours, and thus is distributed.

Remark 3: (i) Different from previous compensators for robust OR or robust bipartite OR of MASs, e.g., [7], [8], [20], where the exosystem output is utilized, here, instead, outputs of agent i and its neighbours and state estimation of the exosystem are integrated into compensator (5).

(ii) In contrast with observers in [18], [19], the observer gain in (6) does not rely on the solution to the linear matrix inequalities. Moreover, if $a_{i0}, a_{ij} \geq 0 (j \in \mathcal{N}_i)$, then observer (6) is reduced to the observer in [3].

Define the estimation error of the distributed observer for agent i as $\tilde{\xi}_i(t) = \xi_i(t) - d_i v(t)$ $(i = 1, \cdots, N)$. Then the following result regarding the estimation error holds.

Theorem 1: If Assumptions (A1) and (A4) hold, then $\lim_{t\to\infty} \tilde{\xi}_i(t) = 0$, $i = 1, \cdots, N$.

Proof: By (2) and (6), one has

$$\dot{\tilde{\xi}}_i = S\tilde{\xi}_i - \gamma\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\text{sgn}(a_{ij})\tilde{\xi}_i - \tilde{\xi}_j) + |a_{i0}|\tilde{\xi}_i\right], \tag{7}$$

where the equality is obtained by the fact that $\text{sgn}(a_{ij})d_i = d_j$. Rewrite (7) as a stack vector form:

$$\dot{\tilde{\xi}} = (I_N \otimes S - \gamma H \otimes I_q)\tilde{\xi},$$

in which $\tilde{\xi} = (\tilde{\xi}_1^T, \tilde{\xi}_2^T, \cdots, \tilde{\xi}_N^T)^T$. Since Assumption (A4) holds, by Lemma 3 in [23], $\text{Re}(\lambda_j(H)) > 0, j = 1, \cdots, N$. According to Assumption (A1), $\text{Re}(\lambda_i(S)) \geq 0, i = 1, \cdots, q$. Thus, by choosing a sufficiently large $\gamma > 0$, one has $\text{Re}\{\lambda_i(S) - \gamma\lambda_j(H)\} < 0, (i = 1, \cdots, q;\; j = 1, \cdots, N)$. Then, $I_N \otimes S - \gamma H \otimes I_q$ is Hurwitz. Therefore, $\lim_{t\to\infty} \tilde{\xi}_i(t) = 0$, $i = 1, \cdots, N$. ■

Remark 4: Different from [20], here only parts of agents can obtain information of the exosystem, and its pinning weight can be negative. So, each agent has to utilize the proposed distributed observer for estimating the exosystem state. From Theorem 1 one can see that the state estimation error of all distributed observers will asymptotically reach zero.

Before deriving the main result, we introduce a matrix property, which is helpful for analyzing the closed-loop system.

Lemma 3: If Assumptions (A1)-(A5) hold, then by choosing K_1, K_2 appropriately, $A_{c1} = \begin{pmatrix} I_N \otimes A + H \otimes BK_1 & I_N \otimes BK_2 \\ H \otimes \Pi_2 C & I_N \otimes \Pi_1 \end{pmatrix}$ is Hurwitz.

Proof: Since Assumption (A4) holds, $\text{Re}(\lambda_i(H)) > 0, i = 1, \cdots, N$. Let $\Omega = U_1^{-1} H U_1$ be the Jordan form of H, where U_1 is a nonsingular matrix. Denote $U = \begin{pmatrix} U_1 \otimes I_N & 0 \\ 0 & U_1 \otimes I_N \end{pmatrix}$. Then

$$\hat{A}_{c1} = U^{-1} A_{c1} U = \begin{pmatrix} I_N \otimes A + \Omega \otimes BK_1 & I_N \otimes BK_2 \\ \Omega \otimes \Pi_2 C & I_N \otimes \Pi_1 \end{pmatrix}. \tag{8}$$

Implementing elementary row and column transformations to $\hat{A}_{c1}$, one obtains

$$V^{-1}\hat{A}_{c1}V = \begin{pmatrix} J_{\hat{A}_{c1},1} & & \\ & \ddots & \\ & & J_{\hat{A}_{c1},N} \end{pmatrix}, \tag{9}$$

where $J_{\hat{A}_{c1},i} = \begin{pmatrix} A + \lambda_i(H)BK_1 & BK_2 \\ \lambda_i(H)\Pi_2 C & \Pi_1 \end{pmatrix}$, $i = 1, \cdots, N$. Denote $T_i = \begin{pmatrix} I_n & 0 \\ 0 & \lambda_i^{-1}(H) I_{n_z} \end{pmatrix}$. Then, $T_i J_{\hat{A}_{c1},i} T_i^{-1} = \begin{pmatrix} A + \lambda_i(H)BK_1 & \lambda_i(H)BK_2 \\ \Pi_2 C & \Pi_1 \end{pmatrix}$.

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Since Assumption (A5) holds, by Assumptions (A1)-(A3) and Lemma 1, we get that (M, N) is stabilizable. Therefore, Riccati equation $M^T P + PM + I_n - PNN^T P = 0$ has a unique positive semi-definite solution P, and hence $M - \theta NN^T P$ is Hurwitz by Theorem 2 in [24], where $\theta \in \mathbb{C}$ and $\mathrm{Re}\theta \geq 1$. Choose K_1, K_2 such that

$$(K_1, K_2) = -\tau^{-1} N^T P, \tag{10}$$

where τ satisfies $0 < \tau \leq \min_{1 \leq i \leq N}\{\mathrm{Re}(\lambda_i(H))\}$. Then, for $\forall i = 1, \cdots, N$, $M + \lambda_i(H)N(K_1, K_2) = M - \lambda_i(H)\tau^{-1}NN^T P$ is Hurwitz. Notice that $T_i J_{\hat{A}_{c1},i} T_i^{-1} = M + \lambda_i(H)N(K_1, K_2)$. Hence, $J_{\hat{A}_{c1},i}$is Hurwitz. Combining (8) and (9), one gets that A_{c1} is Hurwitz. ■

The main result is then summarized as follows.

Theorem 2: For linear uncertain MAS (1) and exosystem (2), if Assumptions (A1)-(A5) hold, and K_1, K_2 satisfy (10), then by choosing a sufficiently large $\gamma > 0$, MAS (1) can achieve robust bipartite OR with protocol (4).

Proof: Let $x_{c\Delta} = (x^T,\ z^T,\ \xi^T)^T$, where $x = (x_1^T,\ x_2^T,\ \cdots,\ x_N^T)^T$, $z = (z_1^T, z_2^T,\ \cdots,\ z_N^T)^T$, and $\xi = (\xi_1^T, \xi_2^T, \cdots, \xi_N^T)^T$. Then, applying protocol (4) to system (1) yields

$$\dot{x}_{c\Delta} = A_{c\Delta} x_{c\Delta} + B_{c\Delta} v,$$

where $A_{c\Delta} =$

$$\begin{pmatrix} I_N \otimes \bar{A} + H \otimes \bar{B}K_1 & I_N \otimes \bar{B}K_2 & 0 \\ H \otimes \Pi_2 \bar{C} & I_N \otimes \Pi_1 & -H \otimes \Pi_2 F \\ 0 & 0 & I_N \otimes S - \gamma H \otimes I_q \end{pmatrix}, \tag{11}$$

$$B_{c\Delta} = \begin{pmatrix} \bar{E} \\ 0 \\ \gamma H D \mathbf{1}_N \otimes I_q \end{pmatrix}, \bar{E} = (\bar{E}_1^T, \bar{E}_2^T, \cdots, \bar{E}_N^T)^T,$$

$D = \mathrm{diag}(d_1, d_2, \cdots, d_N)$. Let

$$\bar{A}_{c1} = \begin{pmatrix} I_N \otimes \bar{A} + H \otimes \bar{B}K_1 & I_N \otimes \bar{B}K_2 \\ H \otimes \Pi_2 \bar{C} & I_N \otimes \Pi_1 \end{pmatrix}. \tag{12}$$

Since Assumptions (A1)-(A5) hold and K_1, K_2 satisfy (10), by Lemma 3, A_{c1}, the nominal system matrix of $\bar{A}_{c1}$, is Hurwitz. Thus, there is an open neighborhood $\mathcal{U}$ of $\Delta = 0$, where $\bar{A}_{c1}$ is Hurwitz. By Assumptions (A1) and (A4), Theorem 1 indicates that $I_N \otimes S - \gamma H \otimes I_q$ is Hurwitz for a sufficiently large $\gamma > 0$. Combining (11) and (12), one obtains that $\forall \Delta \in \mathcal{U}$, $A_{c\Delta}$ is Hurwitz.

Since Assumption (A5) holds, Assumption (A1) and Lemma 2 imply that there exist matrices Q, R in $\mathcal{U}$, satisfying

$$\begin{cases} QS = \left(I_N \otimes \bar{A} + H \otimes \bar{B}K_1\right) Q + (I_N \otimes \bar{B}K_2)R + \bar{E}, \\ RS = (H \otimes \Pi_2 \bar{C})Q + (I_N \otimes \Pi_1)R - HD\mathbf{1}_N \otimes \Pi_2 F, \\ 0 = (I_N \otimes \bar{C})Q - D\mathbf{1}_N \otimes F. \end{cases} \tag{13}$$

Note that $\tilde{\xi} = \xi - (D\mathbf{1}_N \otimes I_q)v$. Let $\tilde{x} = x - Qv$, $\tilde{z} = z - Rv$, and $X_c = (\tilde{x}^T, \tilde{z}^T, \tilde{\xi}^T)^T$. Then, by (13), $\dot{X}_c = A_{c\Delta} X_c$. Therefore, X_c converges to zero asymptotically in $\mathcal{U}$, which implies that $\lim_{t \to \infty} \tilde{x}(t) = 0$. Denote $e = (e_1^T, e_2^T, \cdots, e_N^T)^T$. By (3), one can get $e = (I_N \otimes \bar{C})\tilde{x} + ((I_N \otimes \bar{C})Q - D\mathbf{1}_N \otimes F)v$. This together with (13) gives $\lim_{t \to \infty} e(t) = 0$. By Definition 1, MAS (1) can achieve robust bipartite OR with protocol (4). ■

Remark 5: Compared with results in [3]–[11], where only cooperative interactions are considered, here cooperation and antagonism coexist. In this context, some agents' outputs will track the reference signal while others' converge to the opposite value. This means that robust bipartite OR is achieved. In particular, robust OR problem is also reached under protocol (4) in this paper if only cooperative interactions exist.

Based on Theorem 2, gain matrices Π_1, Π_2, K_1, K_2 in protocol (4) can be chosen as in Algorithm 1.

Algorithm 1 Solution algorithm of gain matrices

Input: system matrices A, B, C, S.
Output: gain matrices Π_1, Π_2, K_1, K_2.
(i) Calculate the minimal polynomial of S and select Π_1, Π_2 satisfying **A**ssumption (A5) in terms of Definition 2.
(ii) Let $M = \begin{pmatrix} A & 0 \\ \Pi_2 C & \Pi_1 \end{pmatrix}, N = \begin{pmatrix} B \\ 0 \end{pmatrix}$.
(iii) Solve $M^T P + PM + I_n - PNN^T P = 0$ to obtain the positive semi-definite solution P.
(iv) Calculate $H = \mathcal{L} + \bar{T}$ and select τ such that $0 < \tau \leq \min_{1 \leq i \leq N}\{\mathrm{Re}(\lambda_i(H))\}$. Output the gain matrices K_1, K_2 by (10).

IV. Simulation

Consider an MAS with 4 agents in (1) and (2), where

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}, \quad E_i = \begin{pmatrix} 0 & i & 0 \\ 0 & 0 & 0 \\ 0 & 0 & i \end{pmatrix} (i = 1, 2, 3, 4),$$

$$S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad F = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}.$$

Clearly, Assumptions (A1)-(A3) hold. Suppose that the uncertain matrices

$$\Delta A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.01\sin(t) & 0 & 0 \end{pmatrix}, \quad \Delta B = \begin{pmatrix} 0.01\cos(t) \\ 0 \\ 0 \end{pmatrix},$$
$$\Delta C = \Delta E_i = 0.$$

Communication interactions among the 4 agents are represented by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$(see Fig. 1), where $\mathcal{A} = [a_{ij}]$, $a_{21} = 2$, $a_{32} = -3$, $a_{34} = 4$. Moreover, $a_{10} = 1, a_{20} = -1, a_{40} = -2$. Obviously, Assumption (A4) holds.

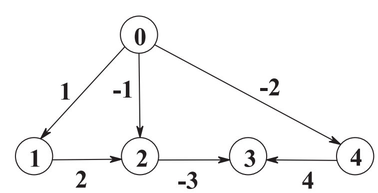

Fig. 1. Communication topology.

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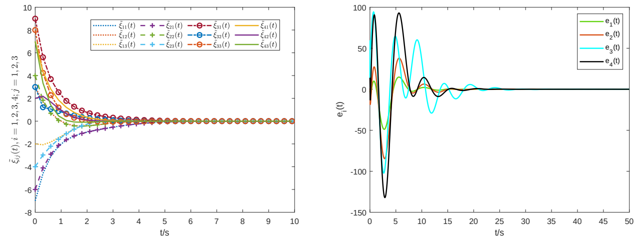
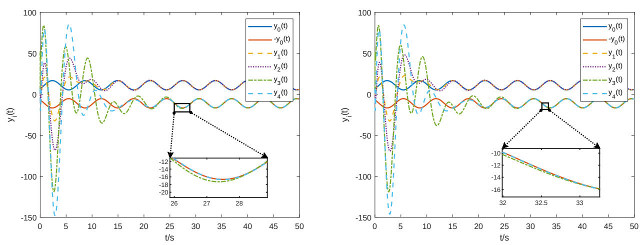


(a) (b)

Fig. 2. (a) The estimation error of distributed observers. (b) The regulated output error.



(a) (b)

Fig. 3. (a) Trajectories of the agent outputs under the state feedback control in [20]. (b) Trajectories of the agent outputs under protocol (4).

By direct calculations, $\lambda_1(H) = 1$, $\lambda_2(H) = 2$, $\lambda_3(H) = 3$, $\lambda_4(H) = 7$. Select $\tau = 1/2$ such that $0 < \tau \leq \min_{1\leq i\leq 4}\{\mathrm{Re}(\lambda_i(H))\} = 1$. By (10), $(K_1, K_2) = (-46.7255, -117.5969, 102.8085, 2.0000, -2.4790, 4.4196)$. Choose $\gamma = 1 > 0$ in protocol (4)-(6), where $d_1 = d_2 = 1$; $d_3 = d_4 = -1$, and $\Pi_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$, $\Pi_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ satisfying Assumption (A5).

As shown in Fig. 2, the estimation error of the observer and the regulated output error of each agent will asymptotically reach zero, respectively. This indicates that robust bipartite OR is achieved under protocol (4). Compared with the state feedback control in [20] using the exosystem output directly, in protocol (4), the state estimation of the exosystem is utilized. Applying these two protocols, we can see from Fig. 3 that the time of agents reaching robust bipartite OR is 28s and 32s, respectively. This implies that the same performance can be achieved even using protocol (4) without the exosystem output.

V. Conclusion

The robust bipartite OR of linear uncertain MASs is addressed over a signed digraph. Since the state and output of the exosystem are unavailable for agents' controllers, an observer-based distributed protocol by exploiting the internal model principle is provided, and sufficient conditions to reach robust bipartite OR are obtained. The proposed control gain depends on the global information of the communication topology. It is meaningful to design controllers for robust bipartite OR in a fully distributed adaptive way. In the future, robust bipartite OR with time delays will be an interesting topic.

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